Springer

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Journal of Mechanical Science and Technology 24 (9) (2010) 1793~1798
www.springerlink.com/content/1738-494x
DOI 10.1007/s12206-010-0627-6

Effect of diffuser angle on the discharge coefficient of miniature critical nozzles †

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(Manuscript received December 10, 2008; revised May 10, 2010; accepted May 21, 2010)

Abstract

Many studies on critical nozzles have been made to accurately measure the mass flow rate of gas and standardize its performance as a flow meter. Recently, much interest has been given to measuring very small mass flow rates in industrial fields, such as MEMS applications. However, the design and performance data of the critical nozzles obtained thus far have been applied mainly to critical nozzles with comparatively large diameters, and available studies on miniature critical nozzles are lacking. In this study, computational fluid dynamics (CFD) method was applied to investigate the influence of the diffuser angle on the discharge coefficient of miniature critical nozzles. In computations, the throat diameter of a critical nozzle varied from 0.2 to 5.0 mm, and the diffuser angle changed from 2° to 8°. The computational results were validated with some available experimental data. The present computational results accurately predicted the discharge coefficient of gas flows through miniature critical nozzles. The discharge coefficient is considerably influenced by the diffuser angle as the throat diameter of the nozzle becomes smaller below a certain value. This implies that miniature critical nozzles should be designed with careful consideration of its effects.

Keywords: Critical nozzle; Compressible flow; Discharge coefficient; Diffuser angle; Reynolds number

1. Introduction

Once gas flow is choked at the nozzle throat, pressure variations downstream of the nozzle have negligible influence on the mass flow. Hence, the discharge coefficient is easily obtained only by determining flow properties upstream of the nozzle. According to the one-dimensional gas dynamics theory, mass flow through a nozzle is given as a function of the pressure and temperature upstream of the nozzle, the nozzle throat diameter, and the specific heat ratio of gas [1]. Critical nozzle is a kind of flow metering device that uses the concept of flow choke, which occurs at the nozzle throat [2]. In addition, it is extensively used to precisely measure the mass flow rate in a variety of industrial facilities.

In the critical nozzle flow, the discharge coefficient is a function of the Reynolds number based on velocity at the nozzle throat and diameter of the nozzle throat [3-5]. At high Reynolds numbers, the discharge coefficient approaches unity, indicating that the one-dimensional, inviscid theory reasonably predicts mass flow. However, at lower Reynolds numbers, the discharge coefficient is considerably lower than unity [6, 7]. In this case, better agreement between the theory and the experiment can be obtained by incorporating into the theory various non-ideal flow mechanisms that affect the actual mass flow. These non-ideal flow mechanisms include viscous effects, multi-dimensional phenomena, and real gas behavior. The presence of viscous effects in the nozzle tends to decrease the mass flow below its ideal value. Physically, the no-slip condition at the nozzle wall results in a layer of slow-moving fluid adjacent to the wall (i.e., the boundary layer). Furthermore, within the boundary layer, the fluid's kinetic energy is irreversibly converted to internal energy (i.e., viscous dissipation) in order for the boundary layer temperature to be larger than the free stream temperature. The larger temperature throughout the boundary layer results in a decrease in the fluid density near the wall. Together, the lower-than-ideal values of fluid velocity and density in the boundary layer result in a reduction in mass from that predicted by the one-dimensional inviscid flow solution.

Multi-dimensional effects result in the curvature of the sonic line, which reduces the actual mass flow relative to the ideal mass flow rate. In multi-dimensional flows (neglecting viscous effects), the sonic line follows an approximately parabolic profile that begins just upstream of the throat along the nozzle wall and extends downstream into the diverging section of the nozzle to its vertex on the centerline. Consequently,

† This paper was recommended for publication in revised form by Associate Editor Do Hyung Lee

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the flow at the throat cross-section will have, in general, a supersonic velocity near the wall and a subsonic velocity near the centerline. On the other hand, the sonic line is for a one-dimensional inviscid flow in order for uniform sonic velocity to exist everywhere along the throat cross-section. Based on the isentropic flow theory, the mass flux obtains its maximum value at a Mach number of unity. Therefore, the mass flow is greater for one-dimensional flow, where sonic conditions exist uniformly across the throat cross-section.

To date, almost all studies on the critical nozzle flow have been mainly focused on viscous effects. Kim et al. [8, 9] have reported the discharge coefficients of a variety of gases in a wide range of Reynolds numbers using computational fluid dynamics (CFD) method. Flow characteristics through the critical nozzle have been well-documented at both considerably low and high Reynolds number regimes. They especially highlighted that the discharge coefficient rationally decreases because of increasing boundary layer, which is expressed by the law of wall, regardless of the Reynolds number. However, their works were limited to critical nozzles with relatively large diameters, wherein their computations predict the gas flow characteristics with good accuracy. In this case, even though viscous effects are significant for predicting the discharge coefficient of the critical nozzle, it is not meaningful to consider multi-dimensional effects because the critical nozzle has nearly the same diffuser angle for measuring the mass flow rate.

However, it may seem that the multi-dimensional effects become significant in the micro-electric mechanical system (MEMS) and nano technology (NT) engineering fields. For instance, a toroidal venturi nozzle with very small diameter is often used to control micro satellite and micro thrust devices, such as micro rocket and micro turbine [10, 11], in which the divergent angle of the nozzle is designed to be large enough to generate high thrust, and the precise measurement of the mass flow rate is essentially important to determine the device performance. Almost all existing data are associated with critical nozzles of large diameters, which are not applicable to such micro devices. In large critical nozzles, the divergent angle of the nozzle has negligible effects on the mass flow measurement [12]. However, in small-size critical nozzles, where Reynolds numbers are very small, viscous wall boundary layer effects are significant, and thickness of the boundary layer significantly increases compared to the flow passage of the critical nozzle. Consequently, multi-dimensional effects should be considered because the distribution of the sonic line can be changed with the diffuser angle.

To date, few studies have documented the effect of the diffuser angle on mass flow rate in small-size critical nozzles [13], so more systematic studies are needed prior to the practical use of small critical nozzles. The objective of this study is to get insight into the flow characteristics of small-size critical nozzles and investigate the effect of diffuser angle on the mass flow rate using the CFD method. The axi-symmeric compressible Navier-Stokes equations are employed to solve gas flow based on a fully implicit finite volume scheme.

Table 1. Nozzle dimensions and flow conditions applied in the present study.

D (mm)	α (degree)	p_o (kPa)	T_o (K)	p_a / p_o
0.2~5.0	2~8	101.3 ~ 202.6	293.15	0.5

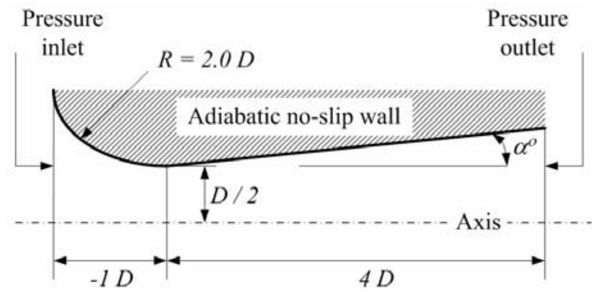


Fig. 1. Computational domain and boundary conditions.

2. Computational analysis

Gas flow, through the critical nozzle, was simulated using the CFD method. The governing equations are given by the conservation forms of mass, momentum, and energy. The axi-symmetric, mass averaged, and time-dependent Navier-Stokes equations, which use a $k-\varepsilon$ turbulent model, were employed in the present computations. Details of the governing equations and computational methodology can be found in refs. 3 and 10.

The critical nozzle employed in the present study is a typical conical type, with its throat diameter D varying from 0.2 mm to 5.0 mm. The convergent part upstream of the throat has a curvature radius of *2.0 D* and length of *1.0 D*, as illustrated in Fig. 1. The divergent part of the nozzle has a half angle of α and is *4.0 D* long. The divergent angle is changed in the range of $2°$ to $8°$, considering a practical design for industrial use.

Pressure inlet and outlet boundary conditions are applied to the nozzle entrance and exit, respectively, where the inlet total pressure (p_o) and back pressure (p_a) of the nozzle are located, respectively. Symmetric conditions are assumed at the axis of the critical nozzle, reducing computational efforts for the full domain. Adiabatic, no-slip conditions are applied to the solid walls.

Table 1 summarizes the flow conditions for the present computations. The nozzle pressure ratio is defined as p_a / p_o, and its value remains constant at *0.5* over the whole computation of the present study. Meanwhile, the inlet total pressure varies in the range of *101.3 kPa* to *202.6 kPa* at a fixed total temperature of $T_0 = 293\ K$. Working gas is assumed to be air that satisfies the law of perfect gas.

Structured grid meshes with about 45,000 grid points are employed. Grids were densely clustered in the boundary layers and near the nozzle throat in order to provide reasonable

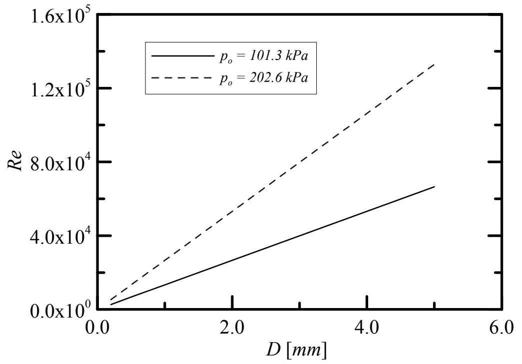


Fig. 2. Variations of Reynolds number with diameter at the nozzle throat.

solutions. Several preliminary computations are performed to ensure grid independent solutions.

3. Results and discussion

The Reynolds number effects of the gas flow through miniature critical nozzles are of practical importance in terms of the discharge coefficient and critical pressure ratio. Fig. 2 presents the relationship between the Reynolds number and nozzle diameters applied in the present study. In this study, the Reynolds number is based on the velocity at the nozzle throat and the diameter of the nozzle throat. For the two pressure values applied, the Reynolds number is given by a linear function of the nozzle throat diameter.

The computed static pressure distributions along the nozzle wall are presented in Fig. 3. The diameter of the critical nozzle is varied from *0.2* to *5.0 mm*, while the diffuser angle is fixed at $\alpha = 4°$. For *D = 5.0 mm*, the flow is accelerated through the nozzle throat at *x/D = 0.0*, and the resulting supersonic flow meets the shock waves near *x/D = 1.3*, leading to a sudden pressure jump. After the pressure jump, the flow is decelerated to a subsonic speed, which should be compromised of the boundary conditions imposed at the nozzle exit. The pressure jump moves upstream along with reduced magnitude as *D* decreases. For *D = 0.2 mm*, the pressure jump is not found in the computed static pressure distribution. Static pressure at the nozzle throat increases slightly as *D* decreases. This is due to the effects of the wall boundary layer on the diameter of the nozzle.

Fig. 4 shows the effect of the diffuser angle on static pressure distributions for two cases: *D = 0.2* and *2.0 mm*. It is likely that flow will more rapidly expand near the nozzle throat as the diffuser angle increases, leading to flow choking further upstream. For $\alpha = 8°$, the static pressures just downstream of the nozzle throat almost do not vary with the distance *x/D*. This is due to the flow separation formed in a nozzle with a large divergent angle. From the present results, flow near the nozzle throat is influenced by the diffuser angle. As the diffuser angle increases, flow reaches sonic conditions further upstream even while the nozzle pressure ratio remains constant. This implies that the mass flow rate through the

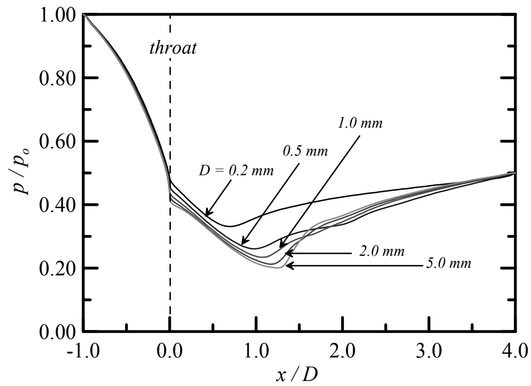


Fig. 3. Static pressure distributions along the nozzle upper wall ($\alpha = 4°$).

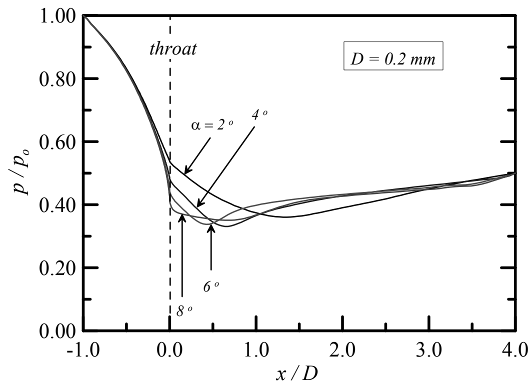


Fig. 4. Static pressure distributions along the nozzle upper wall (*D = 0.2 mm).*

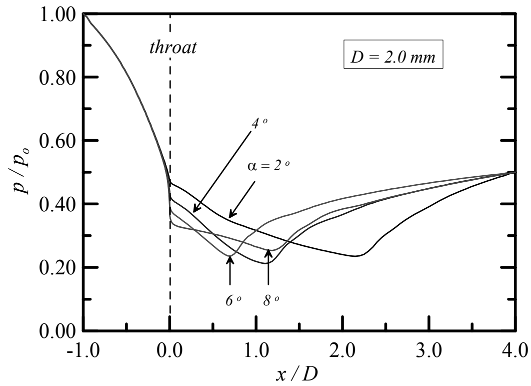


Fig. 5. Static pressure distributions along the nozzle upper wall (*D = 2.0 mm).*

critical nozzle can change with the diffuser angle. A similar trend is also found in Fig. 5, where *D = 2.0 mm*. However, it differs with Fig. 6 in terms of the extent of flow expansion near the nozzle throat. This is because the boundary layer effect is relatively reduced in larger critical nozzles.

Figs. 6 and 7 show the variation of the discharge coefficient *Cd* with the Reynolds number [5-7]. The present computations predicts the measured discharge coefficients for $\alpha = 2°$ well, regardless of the supply pressure level. This clearly shows a general trend in the divergent angle. It seems that, at a given Reynolds number, the discharge coefficient increases as the diffuser angle increases. This tendency appears significant at

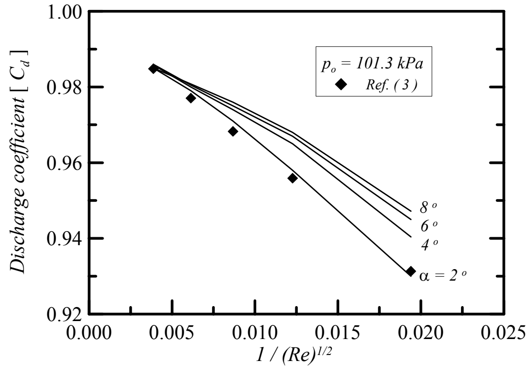


Fig. 6. Relationship between discharge coefficient and Reynolds number ($p_o = 101.3\ kPa$).

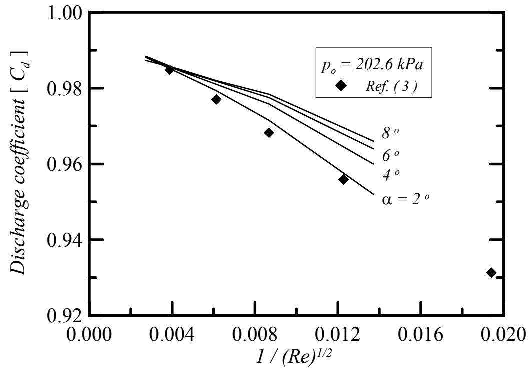


Fig. 7. Relationship between discharge coefficient and Reynolds number ($p_o = 202.6\ kPa$).

low Reynolds numbers but negligible at high Reynolds numbers.

The relationship between the discharge coefficient and nozzle diameter is again presented in Figs. 8 and 9. Regardless of the supply pressure level applied, the discharge coefficient increases along with D, in the range of small D but variation decreases in the range of large D. In particular, the dependence of D on the discharge coefficient seems remarkable in the range of D smaller than $D = 2.0\ mm$, whereas the diffuser angle does not influence the discharge coefficient in the range of large D. The discharge coefficient decreases when the diffuser angle is in the range of small D. The present results imply that the diffuser angle of the critical nozzle should be an important design factor in the application of micro devices.

Fig. 10 shows the effect of the diffuser angle on the boundary layer flow at the nozzle throat. The boundary layer flow appears to be typically turbulent, which can be expressed by both the law of wall and the law of wake. The viscous sub layer was placed in a $y+$ value less than 5.0. Commonly, the discharge coefficient is significantly influenced by the boundary layer because of viscous effects. Based on relevant previous works, development of the boundary layer is known to strongly depend on Reynolds number and nozzle geometry, such as a nozzle curvature. However, the diffuser angles do not affect the development of the boundary layer in Fig. 10. The diffuser angle does not significantly affect the boundary

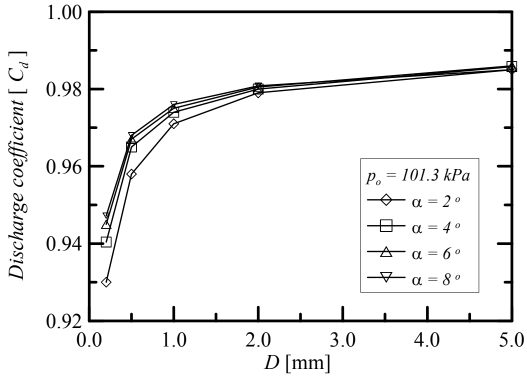


Fig. 8. Influence of the diffuser half angle on the discharge coefficient ($p_o = 101.3\ kPa$).

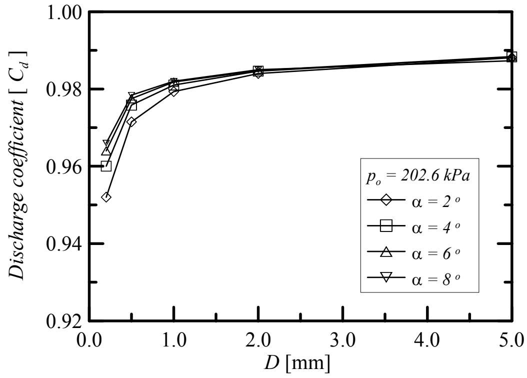


Fig. 9. Influence of the diffuser half angle on the discharge coefficient ($p_o = 202.6\ kPa$).

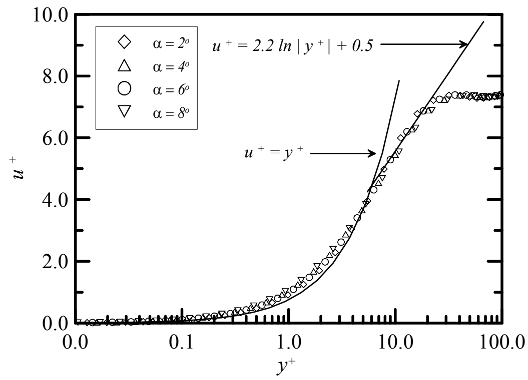
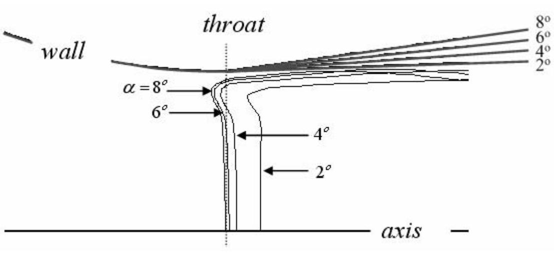


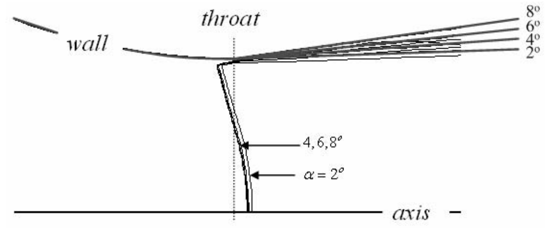
Fig. 10. Boundary layer profiles at the nozzle throat.

layer profiles in the present plot. Therefore, there is a need to find the main reason of the change in mass flow rate with diffuser angle.

In order to scrutinize the flow physics for the reasoning of the diffuser angle effects in the range of small D, Fig. 11 shows the computed sonic line profiles at the nozzle throat. For $D = 0.2\ mm$, at $\alpha = 2°$, the sonic line is placed significantly downstream of the nozzle throat, whereas the sonic line moves upstream as the diffuser angle increases. Such a trend in sonic line locations appears to be relatively insignificant in the case of $D = 2.0\ mm$. The present data indicate that the

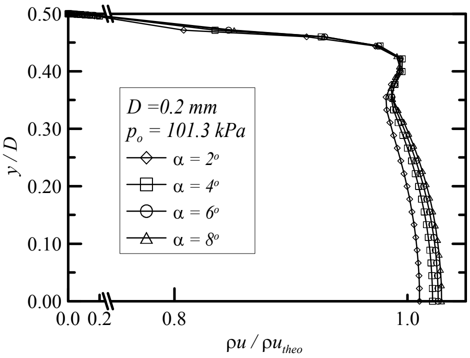


(a) D = 0.2 mm

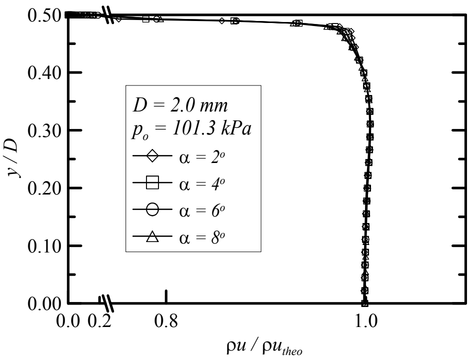


(b) D = 2.0 mm

Fig. 11. Profiles of sonic lines near the throat.



(a) D = 0.2 mm



(b) D = 2.0 mm

Fig. 12. Distributions of normalized mass flux at the throat.

critical nozzle for measuring very small mass flow should be carefully designed with respect to the nozzle diameter and diffuser angle. In order to improve accuracy in small mass flow measurement, further studies should be carried out about the flow control method that can put the sonic line at the nozzle throat.

The computed mass flux profiles at the nozzle throat is shown in Fig. 12, where $p_o = 101.3\ kPa$ and ρu_{theo} is the theoretical mass flux by the isentropic flow assumption. In Fig. 8(a), the effect of the diffuser angle on the mass flux is clearly found in the region below $y/D = 0.4$, which corresponds to outside the wall boundary layer, leading to less mass flux with a decrease in the diffuser angle. The overshoot in the mass flux profiles is observed near the edge of the wall boundary layer. However, the appreciable effect of the diffuser angle is no longer found in larger critical nozzles [Fig. 12(b)], and the overshoot is not formed in the mass flux profiles.

4. Concluding remarks

In this research, a computational study was performed to investigate flow features through miniature critical nozzles. The axi-symmetric compressible Navier-Stokes equations were numerically solved using a fully implicit finite volume method. The two-equation k-ε turbulence closure model was employed to get the turbulent stresses. The diffuser angle of the miniature critical nozzle was varied, together with the nozzle throat diameter. When the diameter of the critical nozzle is smaller than $D = 2.0\ mm$, the discharge coefficient is considerably influenced by the diffuser angle. The larger the diffuser angle becomes, the higher is the discharge coefficient. In the range of such nozzle diameters, the discharge coefficient decreases significantly as the nozzle diameter decreases. This is due to variations in the sonic line location, which appears more significant for small diameters. The result of this study clearly shows that such miniature critical nozzles working at low Reynolds numbers and in micro-devices should be carefully designed with respect to the diffuser angle.

Nomenclature

A^* : Cross-sectional area at nozzle throat
C_d : Discharge coefficient
D : Diameter of nozzle throat
$\dot{m}$: Mass flow rate
p : Static pressure
p_a : Back pressure
p_o : Inlet total pressure
R : Gas constant
Re : Reynolds number
T : Temperature
T_o : Inlet total temperature
x : Distance
y^+ : Non-dimensional distance
u^+ : Non-dimensional velocity
α : Diffuser angle
ρ : Density
ρu : Mass flux
ρu_{theo} : Theoretical mass flux

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